Model Question

B.Sc. Sem-IV Paper- C-10

~~Mod~~ Ring Theory (C 10)

Short answer type question.

1). Define a Ring.
Prove that a) The Zero of a ring R is unique
b) The inverse of a ring is unique.

2) Define
a) Field
b) Integral Domain
c) Division Ring
d) Boolean Ring.

~~3). Prove that~~ a ~~finite integral Domain is a field~~

~~4) Prove that every field is~~ an ~~Integral domain~~

5) If R is a ring such that
$a^2 = a \ \forall a \in R$.
Prove that
i) $a + a = 0 \ \forall \ a \in R$
ii) $a + b = 0 \Rightarrow a = b$
iii) R is a Commutative Ring.

6) Define a simple Ring.
Prove that a division ring is a simple ring.

7). Define
a) An ideal
b) Quotient Ring
c) Ring Homomorphism ~~of~~
d) Kernel of Homomorphism
e) Embedding of a ring

Prove that

8) A ring has no zero divisors iff the Cancellation Law hold in R.

9) Define characteristic of a Ring.
Prove that the characteristic of an integral domain is zero or prime.

10) Let $f: R \longrightarrow R_1$ be a ring homomorphism then P.T.
i) $f(0) = 0$
2) $f(-a) = -f(a) \ \forall \ a \in R$
3) The Kernel of f is an ideal of R.

3) For all a, b in a ring R
i) $a0 = 0a = 0$
ii) $a(-b) = (-a)b = -ab$
iii) $(-a)(-b) = ab$.

4) A non-empty subset S of a ring R is subring of R iff $a-b \in S$ and $ab \in S \ \forall \ a,b \in S$.

Long answer type question

223
1) Let R be a ring and N, an ideal of R then $\frac{R}{N}$ is a ring under the addition and multiplication defined as under:
for $r+N,\ s+N \in \frac{R}{N}$

$$(r+N)+(s+N) = (r+s)+N$$
$$(r+N)(s+N) = rs+N.$$

226
2) An ideal M of a commutative ring R with unity is a maximal ideal iff $\frac{R}{M}$ is a field.

232
3) State and Prove Fundamental Theorem of Homomorphism.

4) a). P.T. a finite integral domain is a field.
b) P.T. ~~a~~ every field is an integral domain.

5) Let $B \subseteq A$ be two ideals of a ring R. then $\frac{R}{A} \cong \frac{R/B}{A/B}$ (First theorem of Isomorphism).

215
6) a). For two ideals A and B of a ring R, $A \cup B$ is an ideal of R iff either $A \subseteq B$ or $B \subseteq A$.

216
b). For any two ideals A and B of a ring R, $A+B = \langle A \cup B \rangle$

7) P.T. a ring can be embedded in a ring with unity